Introductory Chemistry

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Chapter 2

Measurement and Problem Solving

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Modified text by Dr. Bytautas (Galveston College, updated version includes tutorial videos. Jan. 22, 2025)

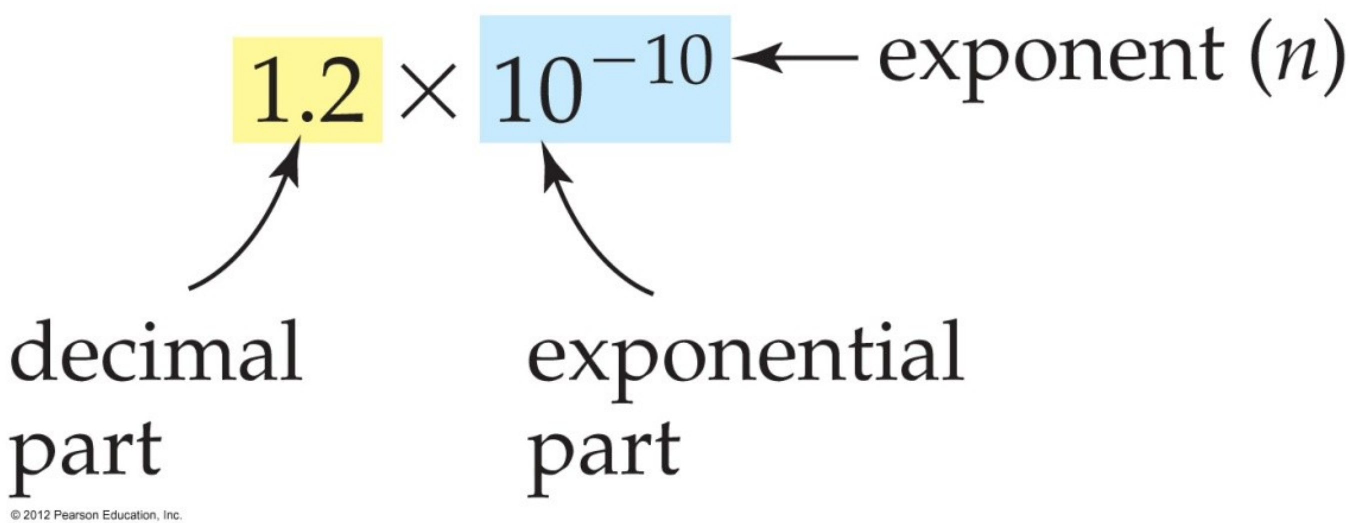
2.1 Measuring Global Temperatures

- Average global temperatures have risen by 0.6 °C in the last century.
- **The uncertainty is indicated by the last reported digit.**
- By reporting a temperature increase of 0.6 °C, the scientists mean 0.6 +/- 0.1 °C.
- The temperature rise could be as much as 0.7 °C or as little as 0.5 °C, but it is not 1.0 °C.
- The degree of certainty in this particular measurement is critical, influencing political decisions that directly affect people's lives.

2.2 Scientific Notation: Writing Large and Small Numbers

- **A number written in <u>scientific notation</u> has two parts.**

- A **<u>decimal part</u>: a number that is between 1 and 10.**
- An **<u>exponential part:</u> 10 raised to an exponent, *n*.**



2.2 Scientific Notation: Writing Large and Small Numbers

- **A positive exponent means 1 multiplied by 10 *n* times.**
- **A negative exponent (–*n*) means 1 divided by 10 *n* times.**

Recommended You tube tutorials:

PRACTICE TUTORIALS:

SCIENTIFIC NOTATION practice problem tutorial (step-by-step-solution):

https://www.khanacademy.org/math/in-in-class-7th-math-cbse/x939d838e80cf9307:exponents-and-powers/x939d838e80cf9307:large-numbers-in-standard-form/v/scientific-notation-old

Writing Very Large and Very Small Numbers

- A positive exponent means 1 multiplied by 10 *n* times.
- A negative exponent (–*n*) means 1 divided by 10 *n* times.

$$10^0 = 1$$

$$10^1 = 1 \times 10 = 10$$

$$10^2 = 1 \times 10 \times 10 = 100$$

$$10^3 = 1 \times 10 \times 10 \times 10 = 1000$$

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10 \times 10} = 0.01$$

$$10^{-3} = \frac{1}{10 \times 10 \times 10} = 0.001$$

To convert a number to scientific notation

- **Move the decimal point to obtain a number between 1 and 10.**

- **Multiply that number (the decimal part) by 10 raised to the power that reflects the movement of the decimal point.**

To convert a number to scientific notation

$$5983 = 5.983 \times 10^3$$

3 2 1

$$0.00034 = 3.4 \times 10^{-4}$$

1 2 3 4

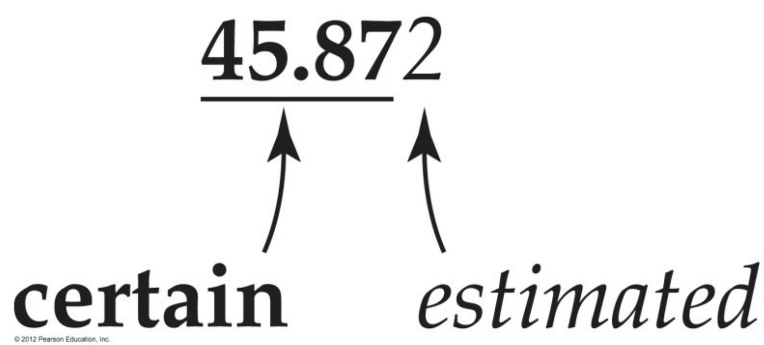
- **If the decimal point is moved to the left, the exponent is <u>positive.</u>**
- **If the decimal is moved to the right, the exponent is <u>negative.</u>**

2.3 Significant Figures:
Writing Numbers to Reflect Precision

Pennies come in whole numbers, and a count of seven pennies **means *seven whole pennies.***

Our knowledge of the amount of gold in **a 10-g gold bar *depends on how precisely it was measured.***

Scientific numbers are reported so that every digit is certain except the last, which is estimated.



The first four digits are certain; the last digit is estimated.

The greater the precision of the measurement, the greater the number of significant figures.

Figure 2.1 Estimating tenths of a gram

- **This balance has markings every 1 g.**
- We estimate to the
- tenths place.
- To estimate between markings, mentally
- divide the space into 10 equal spaces and estimate the last digit.

- **This reading is 1.2 g.**

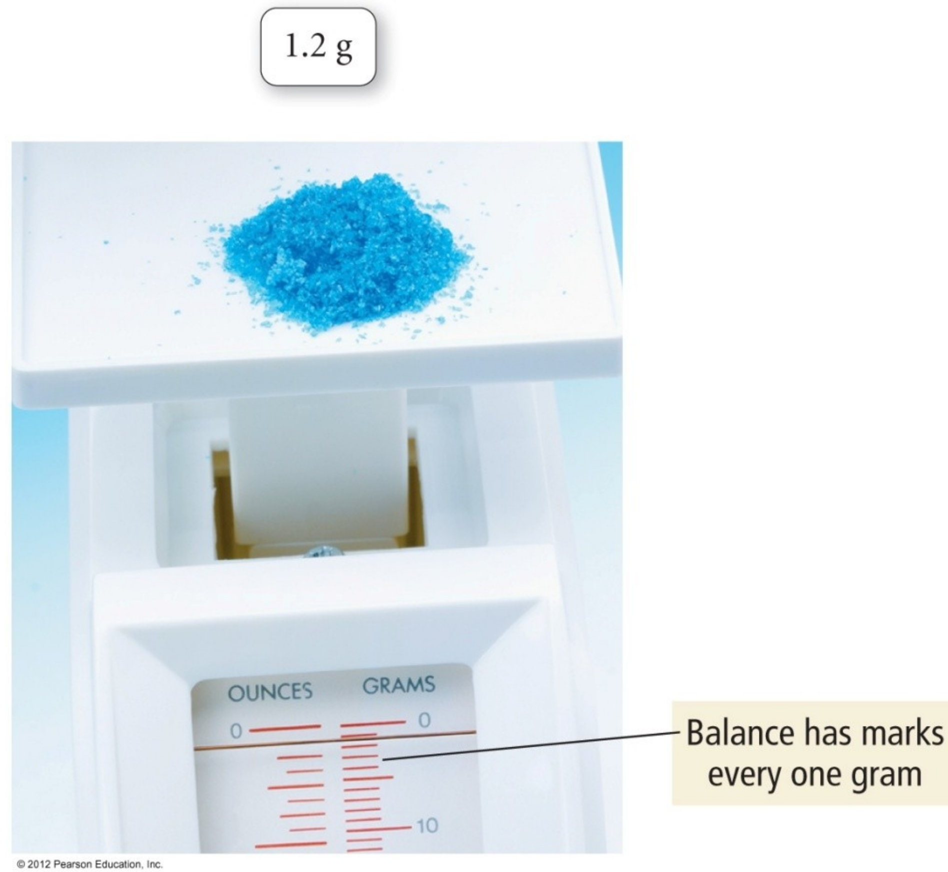
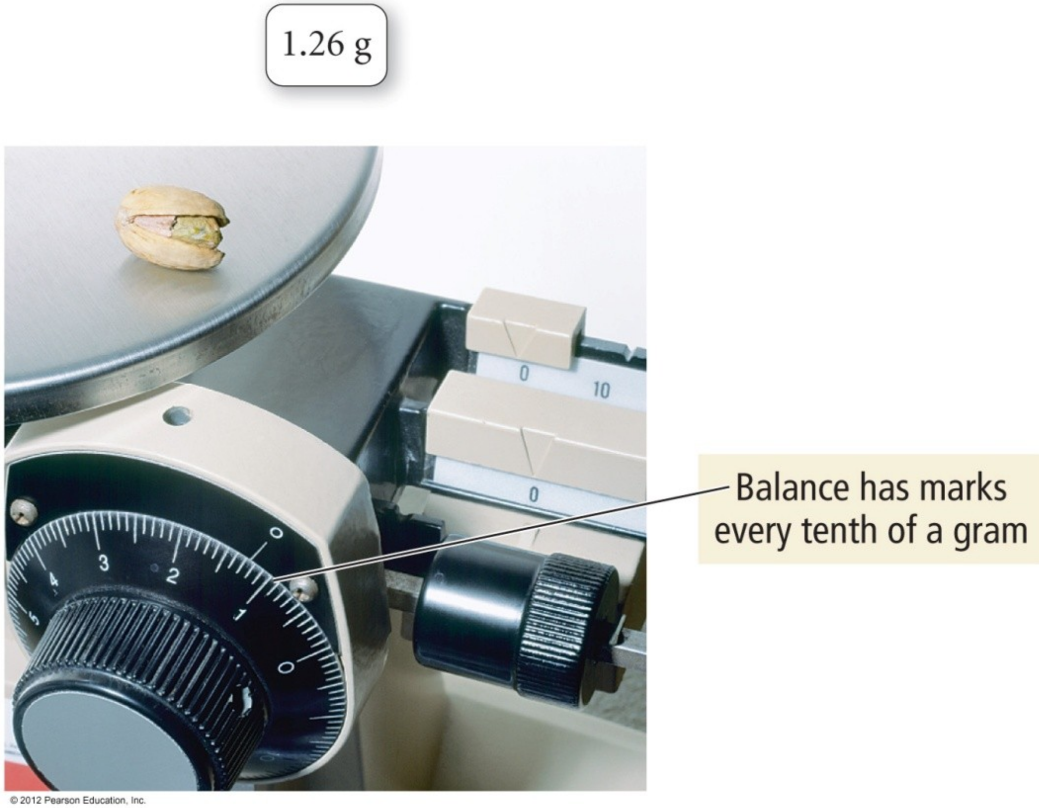


Figure 2.2 **Estimating hundredths of a gram**

- **This scale has markings every**
- **0.1 g.**
- We estimate to the hundredths place.
- The correct reading
- is **1.26 g.**



Counting significant figures in a correctly reported measurement

1. **All nonzero digits are significant.**
2. **Interior zeros** (zeros between two numbers) **are significant.**
3. **Trailing zeros** (zeros to the right of a nonzero number) that fall **after a decimal point are significant.**
4. **Trailing zeros that fall before a decimal point are significant.**
5. **Leading zeros (zeros to the left of the first nonzero number) are NOT significant.** They only serve to locate the decimal point.
6. **Trailing zeros at the end** of a number, but before an implied decimal point, **are ambiguous** and should be avoided.

Exact Numbers:

***Exact numbers** have **an unlimited** number of significant figures.*

- **Exact counting of discrete objects**
- **Integral numbers that are part of an equation**
- **Defined quantities**
- ***Some conversion factors are defined quantities while others are not.***

Identifying Exact Numbers

- *Exact numbers have an unlimited number of significant figures.*
- Exact counting of discrete objects
- Integral numbers that are part of an equation
- Defined quantities
- *Some conversion factors are defined quantities, while others are not.*

1 in. = 2.54 cm exact

How many significant figures are in each number?

0.0035	two significant figures
1.080	four significant figures
2371	four significant figures
2.97×10^5	three significant figures
1 dozen = 12	unlimited significant figures
100.00	five significant figures
100,000	ambiguous(?)

Recommended You tube tutorials:

PRACTICE TUTORIALS:

SIGNIFICANT FIGURES practice problem tutorial (step-by-step-solution):

https://www.khanacademy.org/math/arithmetic-home/arith-review-decimals/arithmetic-significant-figures-tutorial/v/significant-figures

- https://www.khanacademy.org/math/arithmetic-home/arith-review-decimals/arithmetic-significant-figures-tutorial/v/more-on-significant-figures

2.4 Significant Figures in Calculations

Rules for Rounding:

- When numbers are used in a calculation, the result is rounded to reflect the significant figures of the data.

- For calculations involving multiple steps, round only the final answer— do not round off between steps.

  This prevents small rounding errors from affecting the final answer.

2.4 Significant Figures in Calculations

Rules for Rounding:

- **Use only the *last (or leftmost) digit being dropped* to decide in which direction to round—ignore all digits to the right of it.**

- **Round down if the last digit dropped is 4 or less; round up if the last digit dropped is 5 or more.**

2.4 Significant Figures in Calculations

Multiplication and Division Rule:

The result of multiplication or division carries the same number of significant figures as the factor with the fewest significant figures.

2.4 Significant Figures in Calculations

Multiplication and Division Rule:

5.02	$\times$	89.665	$\times$	0.10	=	45.0118	=	45
(3 sig. figures)		(5 sig. figures)		(2 sig. figures)				(2 sig. figures)

The intermediate result (in blue) is rounded to two significant figures to reflect the least precisely known factor (0.10), which has two significant figures.

2.4 Significant Figures in Calculations

Multiplication and Division Rule:

$$5.892 \div 6.10 = 0.96590 = 0.966$$

(4 sig. figures) (3 sig. figures) (3 sig. figures)

The intermediate result (in blue) is rounded to three significant figures to reflect the least precisely known factor (6.10), which has three significant figures.

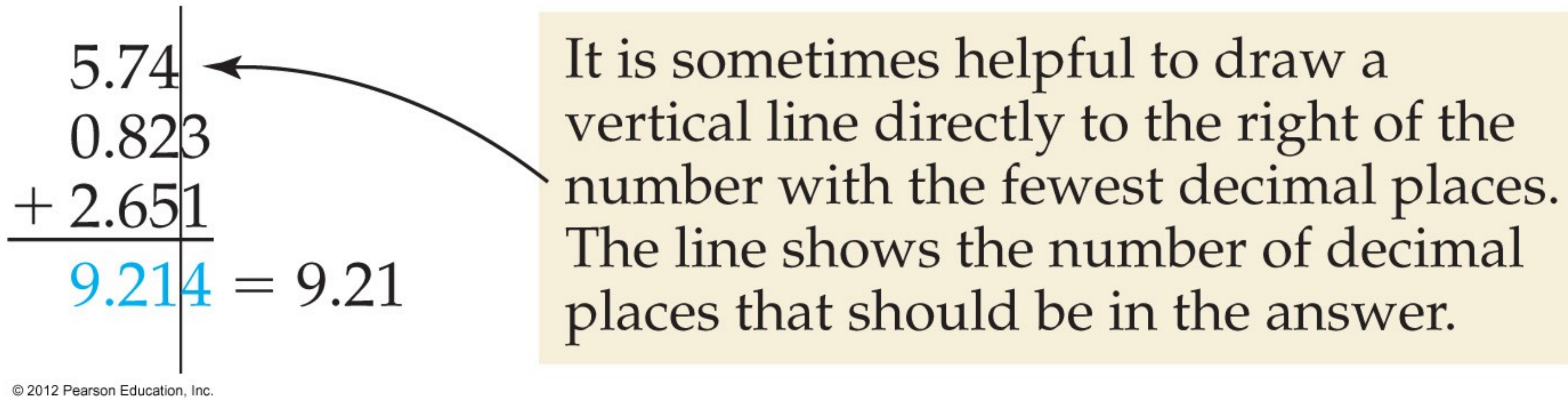
2.4 Significant Figures in Calculations

Addition and Subtraction Rule:

In addition or subtraction calculations, the result carries the same number of decimal places as the quantity carrying the fewest decimal places.

2.4 Significant Figures in Calculations

Addition and Subtraction Rule:



We round the intermediate answer (in blue) to two decimal places because the quantity with the fewest decimal places (5.74) has two decimal places.

2.4 Significant Figures in Calculations

Addition and Subtraction Rule:

$$\begin{array}{r} 4.8 \\ -\ 3.965 \\ \hline 0.835 \end{array} = 0.8$$

We round the intermediate answer (in blue) to <u>one</u> decimal place because the quantity with the <u>fewest decimal places</u> (4.8) has <u>one</u> decimal place.

Calculations Involving Both Multiplication/Division and Addition/Subtraction

In calculations involving both multiplication/division and addition/subtraction,

do the steps in parentheses first;

determine the correct number of significant figures in the intermediate answer without rounding;

then do the remaining steps.

Calculations Involving Both Multiplication/Division and Addition/Subtraction

In the calculation $3.489 \cdot (5.67 - 2.3)$;

do the step in parentheses first. $5.67 - 2.3 = 3.\underline{3}7$

Use the subtraction rule to determine that the intermediate answer has only one significant decimal place.

To avoid small errors, it is best not to round at this point; instead, underline the least significant figure as a reminder.

$$3.489 \times 3.\underline{3}7 = 1\underline{1}.758 = 12$$

Use the multiplication rule to determine that the intermediate answer (11.758) rounds to two significant figures (12) because it is limited by the two significant figures in $3.\underline{37}$.

Recommended You tube tutorials:

PRACTICE PROBLEM TUTORIALS:

MULTIPLYING and DIVIDING with SIGNIFICANT FIGURES practice problem tutorial (step-by-step-solution):

https://www.khanacademy.org/math/arithmetic-home/arith-review-decimals/arithmetic-significant-figures-tutorial/v/multiplying-and-dividing-with-significant-figures

ADDING and SUBTRACTING with SIGNIFICANT FIGURES practice problem tutorial (step-by-step-solution):

- https://www.khanacademy.org/math/arithmetic-home/arith-review-decimals/arithmetic-significant-figures-tutorial/v/addition-and-subtraction-with-significant-figures

Self-Check problems (with solutions)

Express your answer within the correct number of significant digits:

(1) $1459.3 + 9.77 + 4.32 \quad =$

(1) Line-up!

$$
\begin{array}{r}
1459.3 \\
+ \quad 9.77 \\
4.32 \\
\hline
1473.39
\end{array}
$$

round-up

$$\boxed{1473.4}$$

(3) $432 \ + \ 7.3 \ - \ 28.523 \ =$

(3)

$$
\begin{array}{rl}
 & 432. \\
+ & 7.3 \\
- & 28.523 \\
\hline
 & 410.777
\end{array}
$$

$\boxed{411}$ (after round-off)

(5) $(1.7 \times 10^6 \div 2.63 \times 10^5) + 7.33 =$

(5)

$$(1.7\times10^6 \div 2.63\times10^5) + 7.33 =$$

First, do the division in the parantheses.

2SF

$$\frac{1.7\times10^6}{2.63\times10^5} = \frac{1.7}{2.63}\times\frac{10^6}{10^5} =$$

3SF

$$= 0.6463878\times10^1$$

$$= 0.6463878\cdot10^1 = 6.\underline{4}63878$$

underscore!

(2-SF)

Finally, add:

$$\begin{array}{r} 6.463878 \\ +\ 7.33 \\ \hline 13.793878 \end{array}$$

$\boxed{13.8}$ — (after round-off)

2.5 The Basic Units of Measurement

The unit system for science measurements, based on the metric system, is called the **International System** of units (*Système International d'unités*) or ***SI* units**.

TABLE 2.1 Important SI Standard Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature*	kelvin	K

*Temperature units are discussed in Chapter 3.

2.5 Basic Units of Measurement

- The kilogram is a measure of mass, which is different from weight.
- The mass of an object is a measure of the quantity of matter within it.
- The weight of an object is a measure of the gravitational pull on that matter.
- Consequently, weight depends on gravity while mass does not.

Prefix Multipliers

TABLE 2.2 SI Prefix Multipliers

Prefix	Symbol	Multiplier	
tera-	T	1,000,000,000,000	(10^{12})
giga-	G	1,000,000,000	(10^{9})
mega-	M	1,000,000	(10^{6})
kilo-	k	1,000	(10^{3})
deci-	d	0.1	(10^{-1})
centi-	c	0.01	(10^{-2})
milli-	m	0.001	(10^{-3})
micro-	μ	0.000001	(10^{-6})
nano-	n	0.000000001	(10^{-9})
pico-	p	0.000000000001	(10^{-12})
femto-	f	0.000000000000001	(10^{-15})

Prefix Multipliers

- Choose the prefix multiplier that is most convenient for a particular measurement.
- Pick a unit similar in size to (or smaller than) the quantity you are measuring.
- A short chemical bond is about 1.2×10^{-10} m. Which prefix multiplier should you use?
- The most convenient one is probably the picometer. Chemical bonds measure about 120 pm.

Derived Units

- A derived unit is formed from other units.
- Many units of volume, a measure of space, are derived units.
- Any unit of length, when cubed (raised to the third power), becomes a unit of volume.
- Cubic meters (m^3), cubic centimeters (cm^3), and cubic millimeters (mm^3) are all units of volume.

2.6 Problem Solving And Unit Conversions

- **Getting to an equation to solve from a problem statement requires *critical thinking.***
- **No simple formula applies to every problem, yet you can learn problem-solving strategies and begin to develop some chemical intuition.**
- **Many of the problems can be thought of as *unit conversion problems*, where you are given one or more quantities and asked to convert them into different units.**
- **Other problems require the use of *specific equations* to get to the information you are trying to find.**

Converting Between Units

- **Units are multiplied, divided, and canceled like any other algebraic quantities.**
- **Using units as a guide to solving problems is called *dimensional analysis*.**

Always write every number with its associated unit.

Always include units in your calculations, dividing them and multiplying them as if they were algebraic quantities.

Do not let units appear or disappear in calculations. Units must flow logically from beginning to end.

Converting Between Units

- For most conversion problems, we are given a quantity in some units and asked to convert the quantity to another unit. These calculations take the form:

$$\cancel{\text{given unit}} \times \frac{\text{desired unit}}{\cancel{\text{given unit}}} = \text{desired unit}$$

Converting Between Units

- Conversion factors are constructed from any two quantities known to be equivalent.
- Example:

$$\frac{1 \text{ inch}}{2.54 \text{ cm}} = 1$$

$$\frac{2.54 \text{ cm}}{1 \text{ inch}} = 1$$

Both ratios are equal to 1 and can be used to convert between inches (in) and centimeters (cm).

- Conversion factors are constructed from any two quantities **known to be equivalent.**
- We construct the conversion factor by dividing both sides of the equality by 1 in. and canceling the units.

$$2.54\ \text{cm} = 1\ \text{in.}$$

$$\frac{2.54\ \text{cm}}{1\ \text{in.}} = \frac{1\ \cancel{\text{in.}}}{1\ \cancel{\text{in.}}}$$

$$\frac{2.54\ \text{cm}}{1\ \text{in.}} = 1$$

The quantity $\frac{2.54\ \text{cm}}{1\ \text{in.}}$ equal to 1 and can be used to convert between inches and centimeters.

Converting Between Units

- In solving problems, always check if **the final units are correct**, and consider whether or not the magnitude of the answer **makes sense.**

$$44.7\ \cancel{\text{cm}} \times \frac{1\ \text{in.}}{2.54\ \cancel{\text{cm}}} = 17.6\ \text{in.}$$

- **Conversion factors can be inverted** because they are equal to 1 and the inverse of 1 is 1.

Recommended You tube tutorials:

PRACTICE PROBLEM TUTORIALS:

Unit conversion (prefix-unit conversion) practice problem tutorial (step-by-step-solution):

- https://www.khanacademy.org/math/geometry-home/geometry-volume-surface-area/geometry-volume-rect-prism/v/conversion-between-metric-units

- **Why Volume formula makes sense:**
- https://www.khanacademy.org/math/geometry-home/geometry-volume-surface-area/geometry-volume-rect-prism/a/volume-formula-intuition

Convert inches to millimeters

(14) 45.3 inches to millimeters.

Convert inches to millimeters (SOLUTION)

(14) 45.3 in to millimeters (mm)

1 in = 2.54 cm

1 mm = 10^{-3} m

1 cm = 10^{-2} m

Thus,

$$45.3\ \cancel{in} \times \left(\frac{2.54\ \cancel{cm}}{1\ \cancel{in}}\right) \times \left(\frac{10^{-2}\ \cancel{m}}{1\ \cancel{cm}}\right) \times \left(\frac{1\,mm}{10^{-3}\ \cancel{m}}\right)$$

$$= 45.3 \times 2.54 \times 10^{-2-(-3)} = 45.3 \times 2.54 \times 10^{-2+3}$$

$$= 1150.62 \cong \boxed{1.15 \times 10^{3}}\ mm$$

The Solution Map

- **A solution map** **is a visual outline** that shows the strategic route required to solve a problem.

- **For unit conversion, the solution map focuses on units and how to convert from one unit to another.**

We can diagram conversions using a **solution map**.

- The solution map for converting from inches to centimeters is:

$$\frac{2.54\ \text{cm}}{1\ \text{in.}}$$

- The solution map for converting from centimeters to inches is:

$$\frac{1\ \text{in.}}{2.54\ \text{cm}}$$

General Problem-Solving Strategy

- Identify the starting point (the *given* information).
- Identify the end point (what you must *find*).
- Devise a way to get from the starting point to the end point using what is given as well as what you already know or can look up.
- You can use a ***solution map*** to diagram the steps required to get from the starting point to the end point.
- In graphic form, we can represent this progression as

Given → Solution Map → Find

General Problem-Solving Strategy

- **Sort.** Begin by sorting the information in the problem.
- **Strategize.** Create a solution map—the series of steps that will get you from the given information to the information you are trying to find.
- **Solve.** Carry out mathematical operations (paying attention to the rules for significant figures in calculations) and cancel units as needed.
- **Check.**

Recommended You tube tutorials:

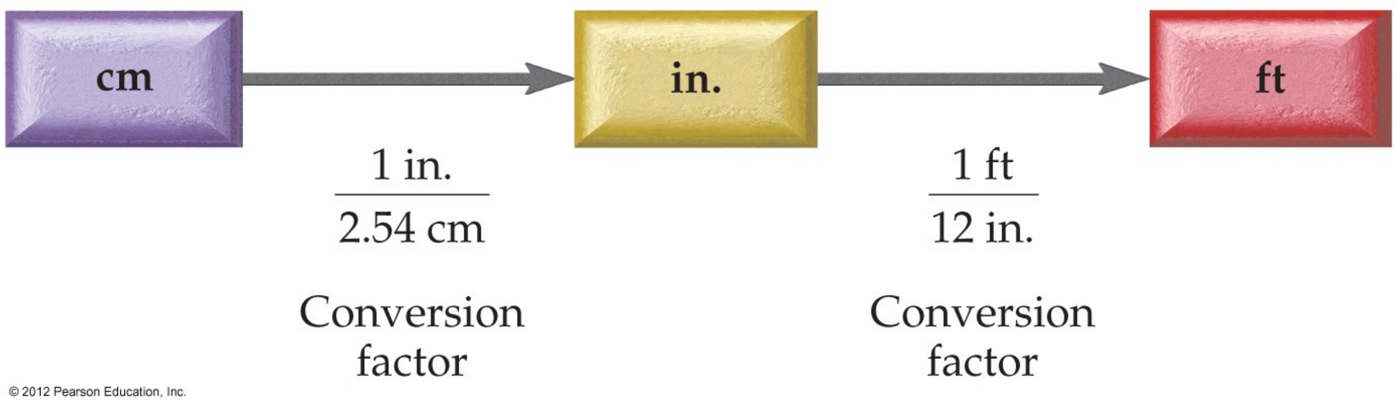
- PRACTICE PROBLEM TUTORIALS:

- Unit conversion practice problem tutorial:
- (by Tyler DeWitt)
- https://www.youtube.com/watch?v=7N0lRJLwpPI

- Unit conversion practice problem with multiple steps tutorial:
- (by Tyler DeWitt)
- https://www.youtube.com/watch?v=LdZ00OFAfaQ

2.7 Solving-Multistep Unit Conversion Problems

- Each step in the solution map should have a conversion factor with the units of the previous step in the denominator and the units of the following step in the numerator.



Once the solution map is complete, follow it to solve the problem.

- **Solution (convert 33.5 cm to ft):**

$$\underline{33.5\ \text{cm}} \times \frac{(1\ \text{in})}{(2.54\ \text{cm})} \times \frac{(1\ \text{ft})}{(12\ \text{in})} = 1.0\underline{9}908\ \text{ft} = 1.10\ \text{ft}$$

Recommended You tube tutorials:

- **Khan academy:**
- **Unit conversion practice problem tutorial:**
- https://www.khanacademy.org/math/cc-fourth-grade-math/imp-measurement-and-data-2/imp-conversion-word-problems/v/multi-step-unit-conversion-examples

- https://www.khanacademy.org/math/cc-fourth-grade-math/imp-measurement-and-data-2/imp-conversion-word-problems/v/multi-step-unit-conversion-examples
- **Unit conversion practice problem with multiple steps tutorial:**
- https://www.khanacademy.org/math/cc-fourth-grade-math/imp-measurement-and-data-2/imp-conversion-word-problems/v/metric-system-unit-conversion-examples

2.8 Units Raised to a Power

When converting quantities with units raised to a power, the conversion factor must also be raised to that power.

Conversion with Units Raised to a Power

We cube both sides to obtain the proper conversion factor.

$$\frac{1\ \text{inch}}{2.54\ \text{cm}} = 1$$

$$\frac{(1\ \text{inch})^3}{(2.54\ \text{cm})^3} = 1$$

QuickTime™ and a
decompressor
are needed to see this picture.

Conversion with Units Raised to a Power

We cube both sides to obtain the proper conversion factor.

$$(2.54\ \text{cm})^3 = (1\ \text{in.})^3$$

$$(2.54)^3\ \text{cm}^3 = 1^3\ \text{in.}^3$$

$$16.387\ \text{cm}^3 = 1\ \text{in.}^3$$

We can do the same thing in fractional form.

$$1255\ \cancel{\text{cm}^3} \times \frac{1\ \text{in.}^3}{16.387\ \cancel{\text{cm}^3}} = 76.5851\ \text{in.}^3 = 76.59\ \text{in.}^3$$

2.9 Density

- Why do some people pay more than $3000 for a bicycle made of titanium?
- For a given volume of metal, titanium has less mass than steel.
- We describe this property by saying that titanium (4.50 g/cm^3) is less dense than iron (7.86 g/cm^3).

Density

The density of a substance is the ratio of its mass to its volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{or} \quad d = \frac{m}{V}$$

Calculating Density

- ***We calculate the density of a substance by dividing the mass of a given amount of the substance by its volume.***

- **For example, a sample of liquid has a volume of**
  22.5 mL and a mass of 27.2 g.

- To find its density, we use the **equation**
  $d = m/V$.

A Solution Map Involving the Equation for Density

- In a problem involving an equation, the solution map shows how the *equation* takes you from the *given* quantities to the *find* quantity.

$$d = \frac{m}{V}$$

Density as a Conversion Factor

- We can use the density of a substance as a conversion factor between the mass of the substance and its volume.

- **For a liquid substance with a density of 1.32 g/cm^3, what volume should be measured to deliver a mass of 68.4 g?**

Density as a Conversion Factor

Solution Map

$$\text{g} \xrightarrow{\frac{1\ \text{cm}^3}{1.32\ \text{g}}} \text{cm}^3 \xrightarrow{\frac{1\ \text{mL}}{1\ \text{cm}^3}} \text{mL}$$

Solution

$$\frac{m}{V} = d$$

$$\frac{m \times \cancel{V}}{\cancel{V} \times d} = \frac{\cancel{d} \times V}{\cancel{d}}$$

$$\frac{m}{d} = V$$

$$\frac{68.4\ \text{g}}{1.32\ \text{g/cm}^3} = 51.8\ \text{cm}^3$$

Measure **51.8 mL** to obtain **68.4 g** of the liquid (note: $1\ \text{mL} = 1\ \text{cm}^3$).

Density as a Conversion Factor

- Table 2.4 provides a list of the densities of some common substances.

- This is useful when solving homework problems.

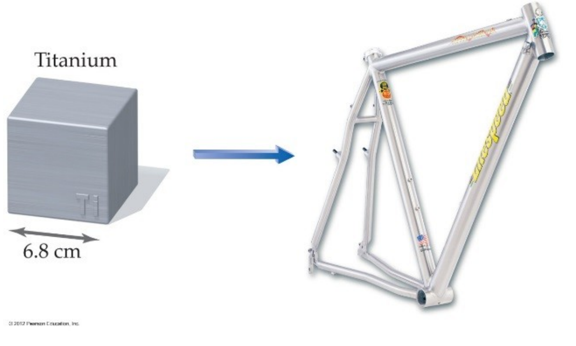
TABLE 2.4 Densities of Some Common Substances

Substance	Density (g/cm^3)
Charcoal, oak	0.57
Ethanol	0.789
Ice	0.92
Water	1.0
Glass	2.6
Aluminum	2.7
Titanium	4.50
Iron	7.86
Copper	8.96
Lead	11.4
Gold	19.3
Platinum	21.4

Example Comparing Densities

- A titanium bicycle frame contains the same amount of titanium as a titanium cube measuring **L =** 6.8 cm on a side.
- Use the density of titanium to calculate the mass in kilograms of titanium in the frame.
- Note that: **Volume (cube) = L^3**

- PROBLEM-1:
- What would be the mass of a similar frame composed of iron?



Recommended You tube tutorials:

- PRACTICE PROBLEM TUTORIALS:

- **Density practice problems (step-by-step solutions)** (by Tyler DeWitt)

- https://www.youtube.com/watch?v=7tVebi3TSsg

- https://www.youtube.com/watch?v=4tYXaCADxfE

Recommended You tube tutorials:

- **PRACTICE PROBLEM TUTORIALS:**

- **Density of the sphere practice problem (step-by-step solution)**

- https://www.khanacademy.org/math/geometry/hs-geo-solids/hs-geo-density/v/volume-density

Calculate the volume V:

(22) What is the volume of an object which has a mass of 3.33 kilograms and density of 11.4 g/cm^3 ?

Calculate the volume V (SOLUTION):

(22)

Given mass = 3.33 kg

density = d = $11.4 \frac{g}{cm^3}$

Find volume = ?

Solution:

$$d = \frac{m}{V} \Rightarrow \boxed{V = \frac{m}{d}}$$

mass (must convert to grams.

$$3.33 \cancel{kg} \times \left(\frac{1000\ g}{1\ \cancel{kg}} \right) = 3.33 \cdot 10^3\ g$$

Substitute to $V = \frac{m}{d}$

$$V = \frac{3.33 \cdot 10^3\ \cancel{g}}{11.4\ \frac{\cancel{g}}{cm^3}} = \boxed{292\ cm^3}$$

Calculate the volume V:

(23) A bar of gold has volume of 5.00 cm^3. Gold has density = 19.3 g/cm^3.

(a) What volume of a cube made from Iron (Iron has density = 7.86 g/cm^3) has exactly the same mass as this bar of gold ?

Calculate the volume V (Solution):

(23)

(23a)

$V = 5.00\ cm^3$

$d_{Au} = 19.3\ \frac{g}{cm^3}$

$d_{Fe} = 7.86\ \frac{g}{cm^3}$

Solution: mass (Au) = mass (Fe)

(23a)

$\Rightarrow m(Au) = d_{Au} \cdot V = 19.3\ \frac{g}{\cancel{cm^3}} \times 5.00\ \cancel{cm^3} = 96.5\ g$

$\Rightarrow$ since $m(Fe) = m(Au) = 96.5\ g$

using $d(Fe) = \frac{m(Fe)}{V(Fe)} \Rightarrow V(Fe) = \frac{m(Fe)}{d(Fe)} = \frac{96.5\ g}{7.86\ \frac{g}{cm^3}} = \boxed{12.3\ cm^3}$